Problem Set 1: The Block-Encoding

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Problem 1 (Block-encodings: tensor products)**.** Let U and V be Q-block encodings of A and B, respectively. Show how to get a Q-block-encoding of $A \otimes B$.

Problem 2 (Extensibility properties)**.** Prove Corollary 1.8 of the lecture notes. Specifically, show that the two extensibility properties allow us to convert a Q-block encoding of A to a dQ-block encoding of $p^{(\mathrm{SV})}(A)$.

Problem 3 (Extensibility properties do not suffice)**.** Let $p(x) = \sum_{k=0}^{d} a_k x^k$ be a polynomial whose coefficients satisfy $\sum |a_k| \leq 1$. Show that $p(x)$ cannot approximate $\sin(100x)$ for any choice of d. That is, show that there is some $x \in [-1, 1]$ such that

$$|p(x) - \sin(100x)| \geq 0.01.$$

Problem 4 (Oblivious amplitude amplification)**.** QSVT is a unifying technique which includes many major quantum algorithms, including amplitude amplification [MRTC21]. In this problem, we show that Oblivious Amplitude Amplification (OAA), as described in [BCCKS17, Lemma 3.6], can be written in our block-encoding framework.

Identify the block-encoding within the aforementioned unitary. What polynomial would effect the same transformation as described in [BCCKS17, Lemma 3.6]?

Remark 1.1. See [Ral20] for more information on how to get block-encodings of density matrices and observables, and how to use this to estimate physical quantities like expectations of Gibbs states. See [BCCKS17] for further discussion of Hamiltonian simulation, placing it in the context of the more general problem of understanding the "fractional query model", "discrete query model", and "continuous query model". See [LC19] (the original paper) or [GSLW19] for a more thorough explanation of the Hamiltonian simulation algorithm.

References

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[Ral20] Patrick Rall. "Quantum algorithms for estimating physical quantities using block encodings". In: *Physical Review A* 102.2 (Aug. 2020), p. 022408. DOI: 10.1103/physreva.102.022408. arXiv: 2004.06832 [quant-ph] (page 1).