Structure learning of Hamiltonians from real-time evolution

Ainesh Bakshi, Allen Liu, Ankur Moitra, Ewin Tang

Motivation

I have an unknown single-qubit unitary U;

how do I estimate U to ε error?
how many uses of U do I need?

Naive: estimate $U|0\rangle$, $U|1\rangle$, $U|+\rangle$ to ε error and reconstruct U.

complexity: $O(1/\varepsilon^2)$ uses of U

But we can do better!

$\Theta(1/\varepsilon)$ uses is necessary and sufficient.
"Heisenberg-limited scaling"

Quantum-mechanical noise in an interferometer

Carlton M. Caves
Phys. Rev. D **23**, 1693 – Published 15 April 1981

Letter | Published: 11 September 2011

A gravitational wave observatory operating beyond the quantum shot-noise limit

The LIGO Scientific Collaboration

Nature Physics **7**, 962–965 (2011) | Cite this article

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Measurement noise 100 times lower than the quantum-projection limit using entangled atoms

Onur Hosten, Nils J. Engelsen, Rajiv Krishnakumar & Mark A. Kasevich

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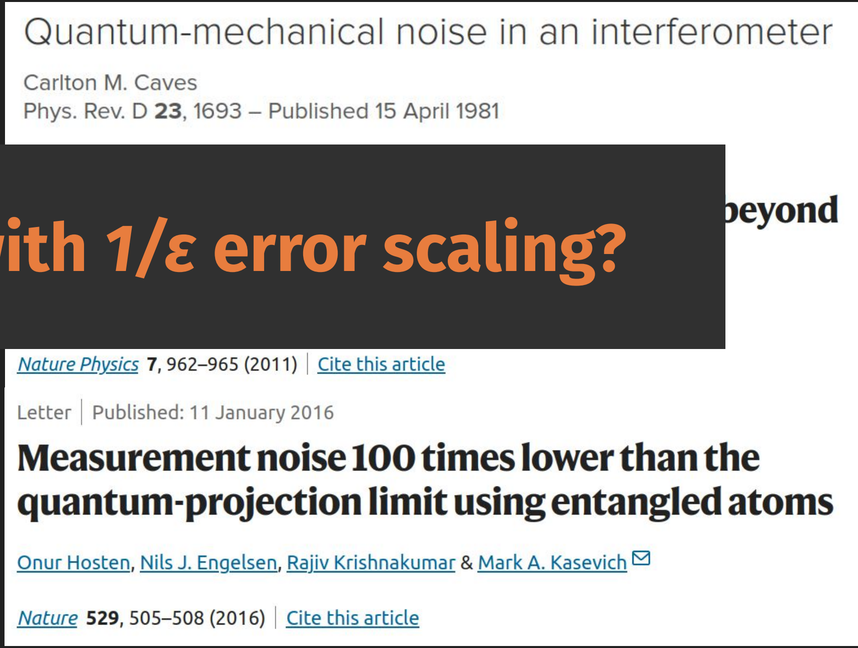
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What can we learn with $1/\varepsilon$ error scaling?

Learning in $1/\varepsilon$ evolution time (the "Heisenberg limit")

To learn a n-qubit unitary U, we need $\Theta(2^{2n}/\varepsilon)$ queries to U. [Haah, Kothari, O'Donnell, T '23]

We want to replicate this, but for tractable classes of unitaries.

"Physically reasonable" choice: evolutions of local Hamiltonians

$U(t) = e^{-iHt}$ for H a local Hamiltonian ($\Theta(n)$ unknown parameters).
NB: "fractional queries" are allowed, $U(t)$ counts for t queries

Can we learn this class with $poly(n)/\varepsilon$ queries?

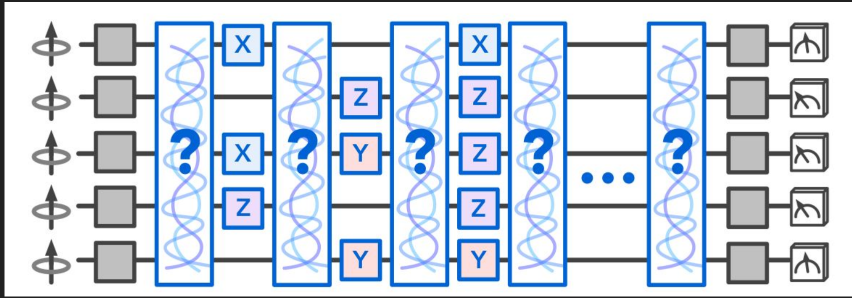
Problem: [HKOT'23] uses algebraic structure of the hypothesis class SU(n)

But **Heisenberg scaling is still possible**!

Learning Hamiltonians with Heisenberg scaling

Given the ability to apply $U(t) = e^{-iHt}$ for a local Hamiltonian H with known terms and unknown parameters, we can learn its parameters in $\Theta(\log(n)/\varepsilon)$ queries. [Huang, Tong, Fang, Su '23]

Technique: dynamical decoupling



But this technique is brittle: it requires

> Knowledge of locality graph
> "Short-range" locality
> A large amount of control: applying $U(t)$ for $t \sim \mathrm{sqrt}(\varepsilon)$

Results

We give a technique for Heisenberg-limited Hamiltonian learning which is much more flexible than dynamical decoupling on several axes.

Our algorithm can:

1. Learn without prior knowledge of the “structure” (i.e. the terms)
2. Smoothly handle long-range terms
3. Learn with only applying $U(t)$ for $t = \Omega(1)$

And it’s (pretty) simple!

See the paper for a detailed comparison with prior work.

Idea: "error amplification" into "term cancellation"

[HKOT'23]: Don't estimate U to ε error;
amplify, then estimate to constant error:

1. Start with an η-good estimate V
2. Estimate the amplified error $E = (UV^\dagger)^\ell$ to constant error ($\ell = \Theta(1/\eta)$)
3. The corrected estimate $(\hat{E})^{1/\ell}V$ is $\eta/2$-good
4. Iterate

We adapt this framework.

Our algorithm:

1. Start with the estimate $V(t) = e^{-iGt}$
2. "Learn the Hamiltonian F" of
   $E = (U(t)V(-t))^\ell$
3. Correct G to $G + F/t\ell$
4. Iterate

Constant-error Hamiltonian learning is easy and works in very broad generality.

Our algorithm inherits the generality of constant-error Hamiltonian learning.

Idea: “error amplification” into “term cancellation”

[HKOT’23]: Don’t estimate U to ε error;

amplify, then estimate to constant error;

1.
2.
3. … d

Key lemma: for small constant t,
E looks like a real-time evolution to the constant-error learning algorithm.

4. Iterate

We adapt this framework.

Our algorithm:

1. Start with the estimate $V(t) = e^{-iGt}$
2. “Learn the Hamiltonian F” of

$$E = (U(t)V(-t))^{\ell}$$

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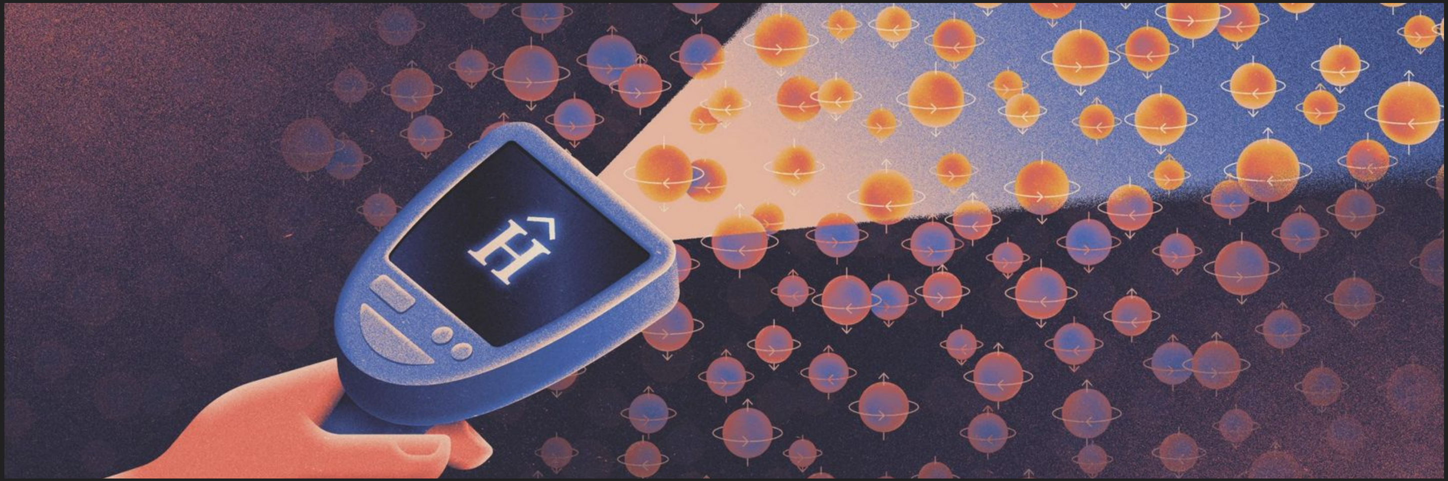
Bonus technique: faster "operator" shadows

Our algorithm is FPT (has no n^k dependence)

FPT "operator shadows": for an observable O, we can estimate the large $tr(OP)$ for low-degree Paulis P without brute-force checking.

Technique: Pauli Goldreich–Levin

Thank you!



credit: Kristina Armitage